STA248 A2 Question 1

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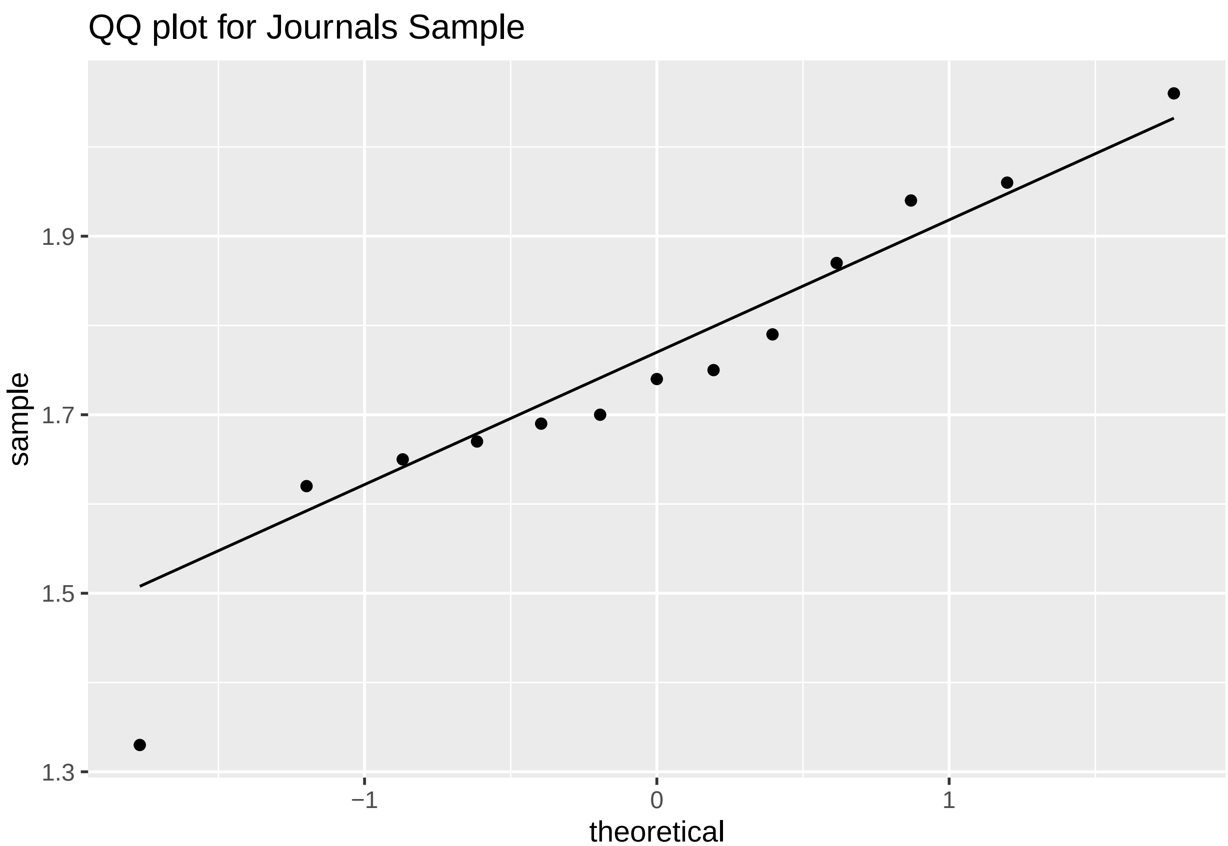
Question 1

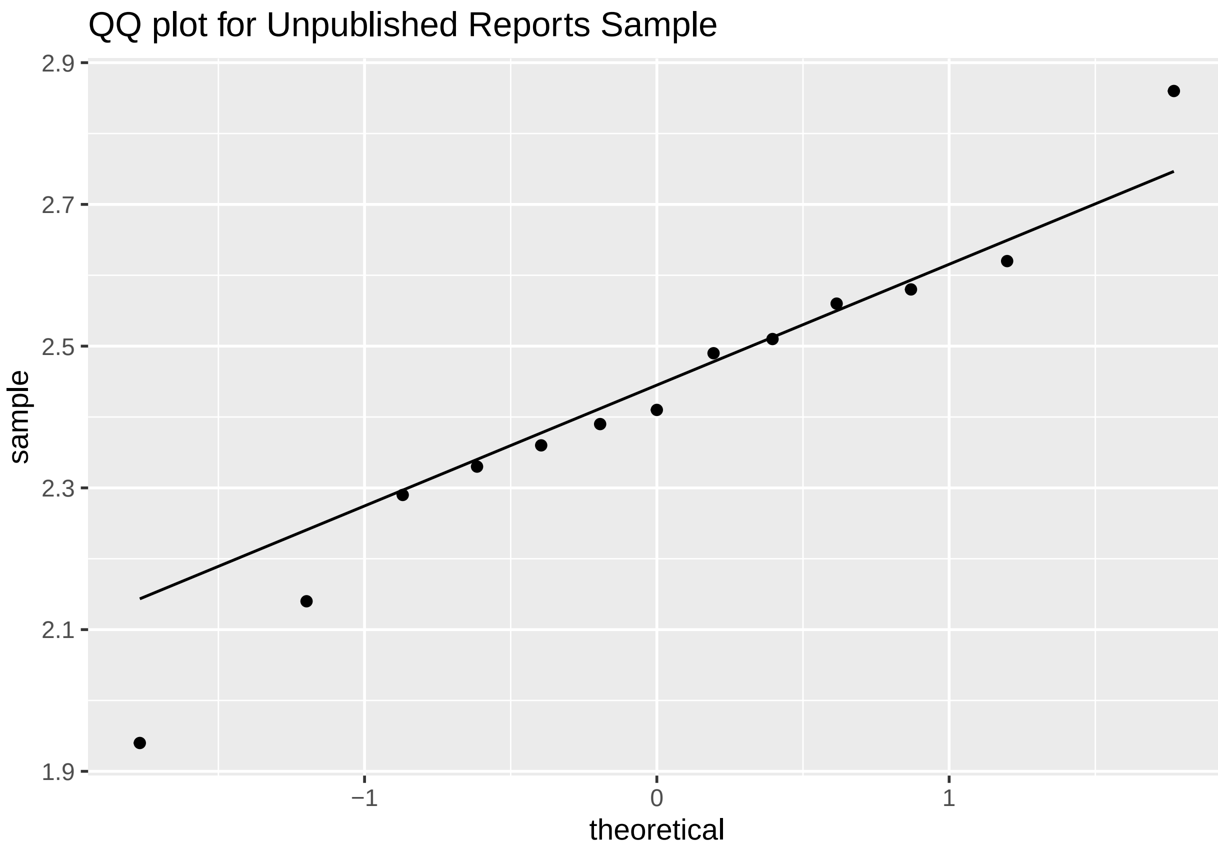
Part a

We will use an F-Test to test for equal variances between the index for confusion of journals and the index of confusion for unpublished reports.

We will denote the random variable for index of confusion of journals as J and similarly for unpublished reports as UR. So it follows that the null hypothesis is $\sigma_J^2 = \sigma_{UR}^2$ and the alternative hypothesis is $\sigma_J^2 \neq \sigma_{UR}^2$. Since we have that the sample sizes of both the journals sample and the unpublished reports sample are both $n = 13$, we know that the degrees of freedom are 12 in both cases.

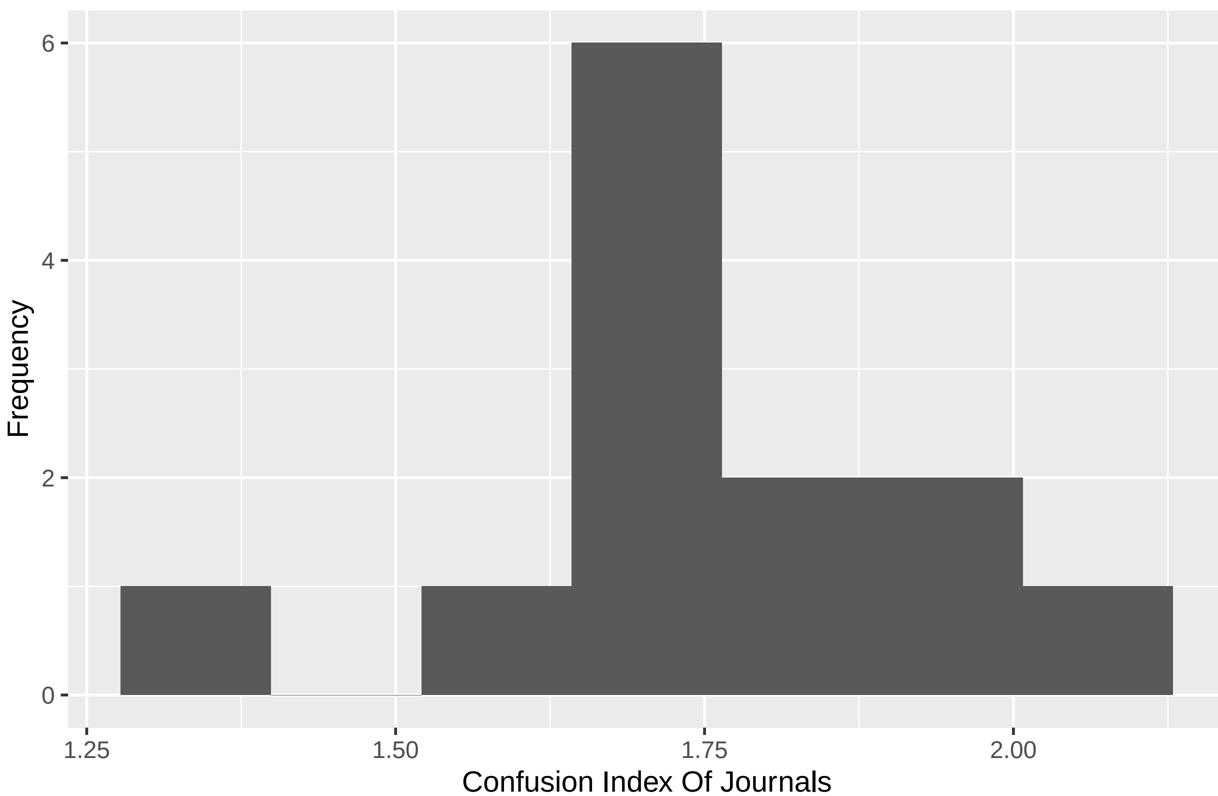
The F-Test assumes that the samples are normal so we will use a qq-plot on both samples to check for normality.



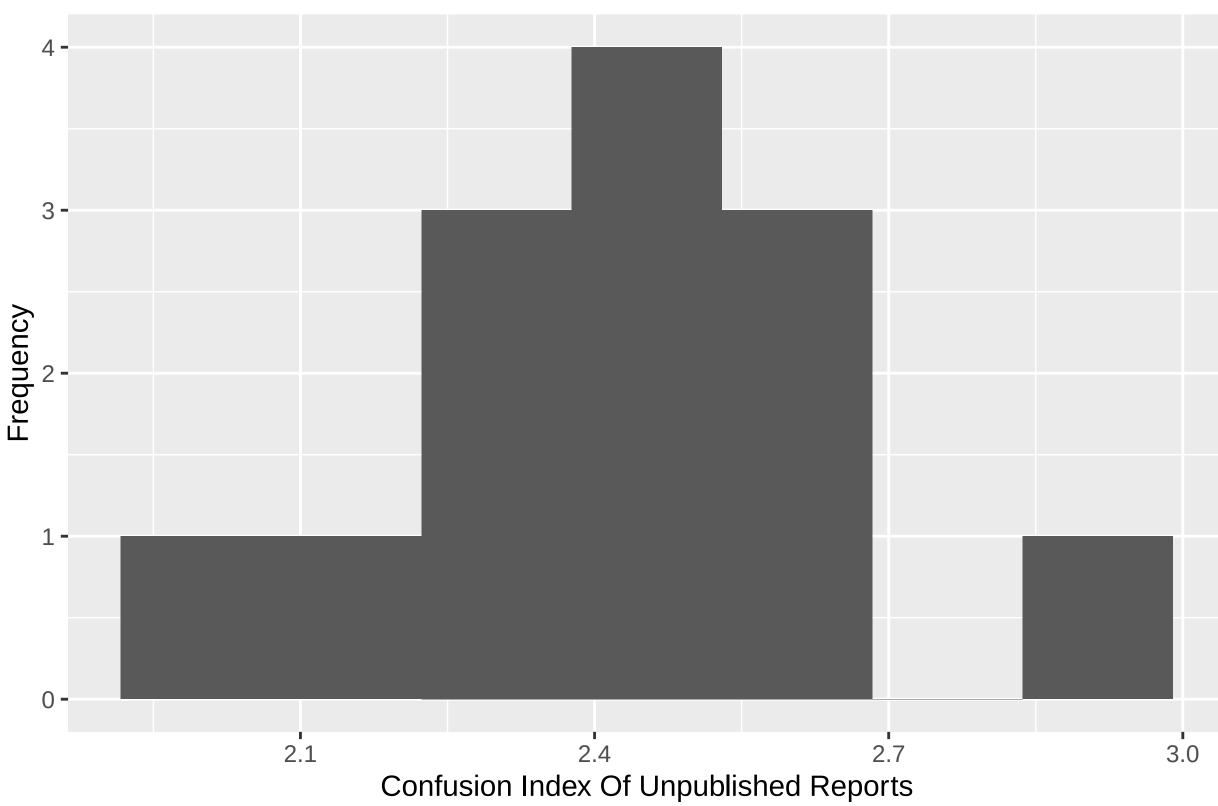


Additionally we will plot the histogram of each sample to visually confirm that it is reasonable to assume normality:

Confusion Index Of Journals



Confusion Index Of Unpublished Reports



Since we see that in both cases the data points follow the line closely and that the histograms are approximately normal, we may assume that both samples are approximately normally distributed. Further, we may assume that the samples are independent of each other since it is reasonable to assume that the writing quality (ie confusion index) is independent of other articles. From here we will generate the sample statistics for each. We get that $s_J^2 = 0.0340141$ and $s_{UR}^2 = 0.0525141$.

We will let F be the test statistic calculated as follows: $0.0525141/0.0340141 = 1.54389209181$, alternatively we can express F as $F\ F_{(12,12)}$. Using the F-table with both of our degrees of freedom to be 12 and the $\alpha = 0.1$ we get the critical value to be 2.15, which is greater than our test statistic of 1.54 so we fail to reject the null hypothesis that $\sigma_J^2 = \sigma_{UR}^2$.

Part b

To see if there appears to be a difference in intelligibility of engineers' English in published journals versus unpublished reports, we will use a two sided t-test for the mean of confusion index. By part a, we were unable to reject the null hypothesis that the variances between the two populations were different, and so we will use the pooled variance in this part.

We will let J be a random variable representing the mean of the confusion index of published (journal) reports while we will let UR be the random variable representing the mean of the confusion index of unpublished reports. Thus, we have the null hypothesis that $\mu_J = \mu_{UR}$ and the alternative hypothesis to be that $\mu_J \neq \mu_{UR}$.

We will now calculate the sample statistics: $\overline{X}_J = 1.751538$ and $\overline{X}_{UR} = 2.421538$, and so the difference between the two sample means is: 0.67.

We also have that by part a the population variances of confusion index of journals and confusion index of unpublished reports are the same, so we have that the pooled variances is: $S_p^2 = \frac{(n_J-1)S_J^2+(n_{UR}^2-1)S_{UR}^2}{n_J+n_{UR}-2} = 0.0432641$. We will therefore also use the $df = n_J + n_{UR} - 2 = 26 - 2 = 24$

Our test statistic will be $P(|\overline{X}_{UR} - \overline{X}_J| \geq 0.67) = P(|T| \geq \frac{0.67}{\sqrt{\frac{0.04}{13}+\frac{0.04}{13}}} = 8.54$. Since we have that 8.54 is far greater than any critical value at any df for $\alpha = 0.05$ for a two sided test, we have sufficient evidence to reject the null hypothesis that there is no difference in the intelligibility of engineers' English in published journals versus unpublished reports, and so we have evidence to support the alternative hypothesis that there is a difference between the intelligibility of engineers' English in published journals versus unpublished reports.

Part c

If we were to construct a 95% confidence interval for the difference in average "index of confusion" scores between published and unpublished reports, we would expect to not see 0 in the interval because 0 would suggest that there is no difference between the true mean of the differences in the average "index of confusion" scores between published and unpublished reports. By part b we see that there is evidence that there is a difference in the average "index of confusion" scores between published and unpublished reports.

Part d

By part b we will use the $df = 24$. Looking at the t table with a two sided significance level of 0.05, we have a t^* critical value of 2.06. Thus we have the standard error of $\sqrt{\frac{0.04}{13} + \frac{0.04}{13}}$ from part b and therefore a margin of error of $t^* \cdot \sqrt{\frac{0.04}{13} + \frac{0.04}{13}} = 0.16159$. So taking our point estimate of 0.67 from part and adding ± 0.16159, we get a 95% confidence interval: (0.5084, 0.8316).